

## Spacetime Symmetry Violation\*

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### Abstract

Supersymmetric models with Lorentz violation can be formulated in superspace. Two theories based on the Wess-Zumino model are discussed. A compactification of superspace can be employed to understand the chiral superfield that arises in the models.

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## I. INTRODUCTION

Spacetime symmetries have played an important part in our understanding of fundamental physics. The discovery of special relativity as well as the Lorentz symmetry underlying it was followed by the proposal for a larger spacetime symmetry, namely supersymmetry. In this case experimental observations require that if spacetime supersymmetry is relevant for describing particle physics, then it must be broken. More recently there has been extended discussion of extra dimensions. If the extra dimensions are compactified, then there is necessarily a violation of the extended Lorentz symmetry that applies to the extra dimensions. This history should encourage us to consider possible connections between these various broken spacetime symmetries (and the various scales involved in the breaking) as well as consider the possibility that the four-dimensional Lorentz symmetry is itself violated even though there is at present no experimental evidence for it. In the following we discuss supersymmetric models based on the Wess-Zumino model that contain Lorentz violation.

## II. SUPERSPACE TRANSFORMATIONS

The Wess-Zumino model [1] can be formulated in terms of differential operators that act on the superfields defined over a superspace of coordinates

$$z^M = (x^\mu, \theta^\alpha, \bar{\theta}_{\dot{\alpha}}) . \quad (1)$$

where  $x^\mu$  are commuting spacetime coordinates and  $\theta^\alpha$  and  $\bar{\theta}_{\dot{\alpha}}$  are anticommuting two-component Weyl spinors. Let

$$X \equiv (\theta\sigma^\mu\bar{\theta})\partial_\mu , \quad (2)$$

so that

$$U_x \equiv e^{iX} = 1 + i(\theta\sigma^\mu\bar{\theta})\partial_\mu - \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\square . \quad (3)$$

Application of  $U_x$  to a superfield  $\mathcal{S}$  produces a coordinate shift  $x^\mu \rightarrow y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$ ,

$$U_x\mathcal{S}(x, \theta, \bar{\theta}) = \mathcal{S}(y, \theta, \bar{\theta}) . \quad (4)$$

A *chiral* superfield is a function of  $y^\mu$  and  $\theta$ ,

$$\Phi(x, \theta, \bar{\theta}) = \phi(y) + \sqrt{2}\theta\psi(y) + (\theta\theta)\mathcal{F}(y) , \quad (5)$$

so it can be expressed in terms of a superfield  $\Psi$  in the following manner  $\Phi(x, \theta, \bar{\theta}) = U_x\Psi(x, \theta)$  where  $\Psi$  depending on only  $x^\mu$  and  $\theta$  and not  $\bar{\theta}$ . The Wess-Zumino model can be expressed as

$$\int d^4\theta \left[ U_x^*\Psi(x, \bar{\theta})^* \right] [U_x\Psi(x, \theta)] + \int d^2\theta \left[ \frac{1}{2}m\Phi^2 + \frac{1}{3}g\Phi^3 + h.c. \right] . \quad (6)$$

The Lorentz-violating extensions [2] of the Wess-Zumino model can be understood in an analogous way as Lorentz-violating transformations on the superfields similar to the one in Eqn. (4). Considering the derivative operator in Eqn. (2), define

$$Y \equiv k_{\mu\nu}(\theta\sigma^\mu\bar{\theta})\partial^\nu, \quad (7)$$

$$K \equiv k_\mu(\theta\sigma^\mu\bar{\theta}), \quad (8)$$

so that

$$U_y \equiv e^{iY} = 1 + ik_{\mu\nu}(\theta\sigma^\mu\bar{\theta})\partial^\nu - \frac{1}{4}k_{\mu\nu}k^{\mu\rho}(\theta\theta)(\bar{\theta}\bar{\theta})\partial^\nu\partial_\rho, \quad (9)$$

$$T_k \equiv e^{-K} = 1 - k_\mu(\theta\sigma^\mu\bar{\theta}) + \frac{k^2}{4}(\theta\theta)(\bar{\theta}\bar{\theta}). \quad (10)$$

Terms necessarily appear that are quadratic in the Lorentz-violating coefficients  $k_{\mu\nu}$  and  $k_\mu$ . Since  $Y$ , like  $X$ , is a derivative operator, the action of  $U_y$  on a superfield  $\mathcal{S}$  is a coordinate shift. On the other hand,  $T_k$  is not a derivative operator and its action does not shift the spacetime coordinate. Consequently the application of these operators is a generalization of the conventional coordinate shift  $x^\mu \rightarrow y^\mu$  usually associated with chiral superfields. The following properties are satisfied:  $U_x^* = U_x^{-1}$ ,  $U_y^* = U_y^{-1}$  and  $T_k^* = T_k$ .

A first supersymmetric model with Lorentz-violating terms can be expressed in terms of a new superfield [3],

$$\Phi_y(x, \theta, \bar{\theta}) = U_y U_x \Psi(x, \theta). \quad (11)$$

Applying  $U_y$  to the chiral and antichiral superfields merely effects the substitution  $\partial_\mu \rightarrow \partial_\mu + k_{\mu\nu}\partial^\nu$ . The chiral superfield  $\Phi_y$  is a function of the variables  $x_+^\mu = y^\mu + ik^{\mu\nu}\theta\sigma_\nu\bar{\theta} = x^\mu + i\theta\sigma^\mu\bar{\theta} + ik^{\mu\nu}\theta\sigma_\nu\bar{\theta}$  and  $\theta$  analogous to how, in the usual case,  $\Phi$  is a function of the variables  $y^\mu$  and  $\theta$ . The Lagrangian is given in terms of integrals over superspace

$$\begin{aligned} & \int d^4\theta \Phi_y^* \Phi_y + \int d^2\theta \left[ \frac{1}{2}m\Phi_y^2 + \frac{1}{3}g\Phi_y^3 + h.c. \right] \\ &= \int d^4\theta [U_y^* \Phi^*] [U_y \Phi] + \int d^2\theta \left[ \frac{1}{2}m\Phi^2 + \frac{1}{3}g\Phi^3 + h.c. \right]. \end{aligned} \quad (12)$$

A superfield appropriate for a second supersymmetric model with Lorentz violation has the form

$$\Phi_k(x, \theta, \bar{\theta}) = T_k U_x \Psi(x, \theta), \quad (13)$$

The superspace integral

$$\int d^4\theta \Phi_k^* \Phi_k = \int d^4\theta \Phi^* e^{-2K} \Phi, \quad (14)$$

describes a CPT-violating model. The  $(\theta\theta)(\bar{\theta}\bar{\theta})$  component of  $\Phi_k^* \Phi_k$  transforms into a total derivative.

### III. SUPERMANIFOLDS

It is clear that compactification of spacetime results in violations of the Lorentz symmetry<sup>1</sup>. Compactification of extra dimensions (those beyond the conventional four) inevitably leads to violations of the Lorentz group that is extended to those extra dimensions. Furthermore models of the Scherk-Schwarz variety [5–7] result in broken supersymmetry in four dimensions from compactification of extra dimensions.

In complexified superspace [8] one can understand the chiral superfields as those defined after a suitable compactification of a supermanifold [9]. Under the transformation

$$(x^\mu, \theta, \bar{\theta}) \rightarrow (x^\mu + i\theta\sigma^\mu\bar{\eta}, \theta, \bar{\theta} - \bar{\eta}) , \quad (15)$$

a chiral superfield is invariant. If one mods out by the discrete subgroup of transformations where the components of  $\eta$  are complex Grassman integers  $m + in$  for integers  $m$  and  $n$ , then only those superfields which are invariant under the transformation in Eqn. (15) are defined on the quotient space [9] since a superfield must be constant along a compact direction. It is straightforward to understand a Lorentz-violating extension of the Wess-Zumino model as a compactification that does not respect the Lorentz symmetry. The relevant transformation on superspace would be

$$(x^\mu, \theta, \bar{\theta}) \rightarrow (x^\mu + i\theta\sigma^\mu\bar{\eta} + ik^{\mu\nu}\theta\sigma_\nu\bar{\eta}, \theta, \bar{\theta} - \bar{\eta}) . \quad (16)$$

Forming the quotient space in the same way results in a chiral superfield that depends on the invariant variables  $x^\mu_+$  and  $\theta$ . Clearly the hope is to link Lorentz violation with supersymmetry breaking in this way.

### IV. CONCLUSIONS

The Wess-Zumino model and two Lorentz-violating extensions of it can be described in terms of transformations on superfields and projections arising from superspace integrals. A geometric interpretation is available by considering a compactification of a complexified superspace.

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<sup>1</sup>An elementary example of the physical implications of the global structure of spacetime is a modified twin paradox. It is only necessary to consider one space and one time dimension. In this case, the twins are on a cylinder  $R \times S_1$  and the compactification picks out a preferred frame [4].

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